The "Figure Eight" Slope

This is the relation equation, pasted from Wikipedia:

```
(%i1) x ^ 4 = a ^ 2 · ( x ^ 2 − y ^ 2 ) ;
```

$$(\%o1) \quad x^4 = a^2 \left(x^2 - y^2\right)$$

We preform implicit differentiation, taking the derivatives of both sides:

```
(%i2) 4 · x ^ 3 = 2 · a ^ 2 · x − a ^ 2 · ' diff ( y ^ 2 , x ) ;
```

$$(\%o2) \quad 4x^3 = 2a^2 x - a^2 \left(\frac{d}{dx} y^2\right)$$

We apply the chain rule, which states that

```
(%i3) ' diff ( z , x ) = ' diff ( z , y ) · ' diff ( y , x ) ;
```

$$(\%o3) \quad \frac{d}{dx} z = \left(\frac{d}{dx} y\right) \left(\frac{d}{dy} z\right)$$

```
(%i4) 4 · x ^ 3 = 2 · a ^ 2 · x − a ^ 2 · ' diff ( y ^ 2 , y ) · ' diff ( y , x ) ;
```

$$(\%o4) \quad 4x^3 = 2a^2 x - a^2 \left(\frac{d}{dx} y\right) \left(\frac{d}{dy} y^2\right)$$

```
(%i5) 4 · x ^ 3 = 2 · a ^ 2 · x − a ^ 2 · 2 · y · ' diff ( y , x ) ;
```

$$(\%o5) \quad 4x^3 = 2a^2 x - 2a^2 y \left(\frac{d}{dx} y\right)$$

Solving for dy/dx, we have the slope.

```
(%i6) ' diff ( y , x ) = ( 2 · a ^ 2 · x − 4 · x ^ 3 ) / ( 2 · a ^ 2 · y ) ;
```

$$(\%o6) \quad \frac{d}{dx} y = \frac{2a^2 x - 4x^3}{2a^2 y}$$

If we assume a = 1, then it simplifies to

```
(%i7) ' diff ( y , x ) = ( x − 2 · x ^ 3 ) / y ;
```

$$(\%o7) \quad \frac{d}{dx} y = \frac{x - 2x^3}{y}$$

For visual pleasure, we would like to plot the figure-eight with a=1, at a point

x=-1/2. Solving the original relation for y, we have...

```
(%i8) f ( x ) : = sqrt ( x ^ 2 − x ^ 4 ) ;
```

$$(\%o8)\quad \mathrm{f}(x) := \sqrt{x^2 - x^4}$$

```
(%i9) f_2 ( x ) : = − sqrt ( x ^ 2 − x ^ 4 ) ;
```

$$(\%o9)\quad f_2(x) := -\sqrt{x^2 - x^4}$$

```
(%i10) f ( − 1 / 2 ) ;
```

$$(\%o10)\quad \frac{\sqrt{3}}{4}$$

So, we plug in point

```
(%i11) ' ( − 1 / 2 , sqrt ( 3 ) / 4 ) ;
```

$$(\%o11)\quad \left(-\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$$

into the point-slope form

```
(%i12) y − y_1 = m · ( x − x_1 ) ;
```

$$(\%o12)\quad y - y_1 = m\,(x - x_1)$$

giving us

```
(%i13) g ( x ) : = − x / sqrt ( 3 ) + 1 / ( 4 · sqrt ( 3 ) ) ;
```

$$(\%o13)\quad \mathrm{g}(x) := \frac{-x}{\sqrt{3}} + \frac{1}{4\sqrt{3}}$$

and now the graph...

```
(%i14) wxplot2d ( [ f , f_2 , g ] , [ x , − 2 , 2 ] , [ y , − 2 , 2 ] ) $
```

plot2d: expression evaluates to non-numeric value somewhere in plotting range.
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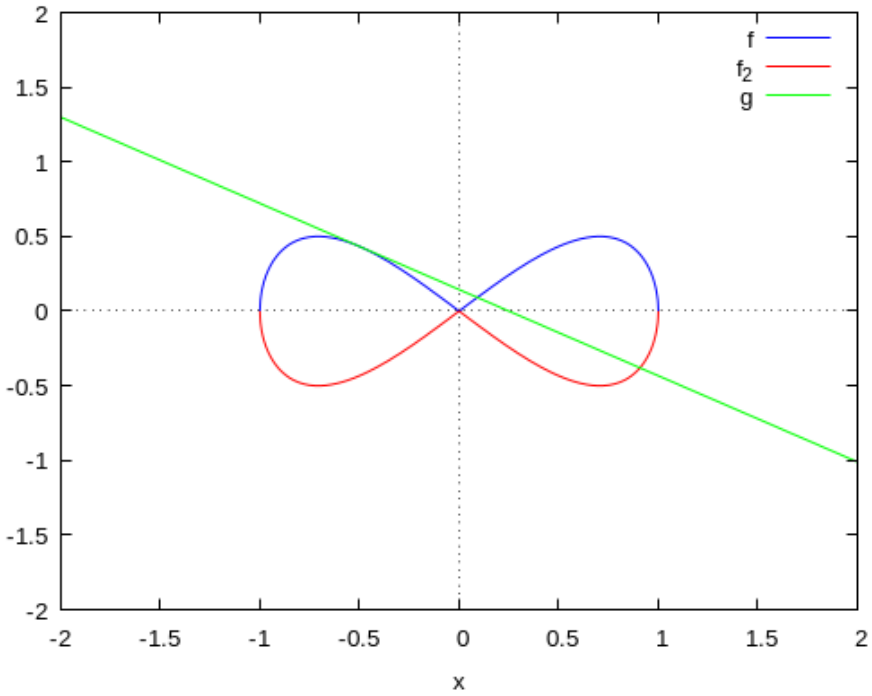
(%t14)



Created with wxMaxima.